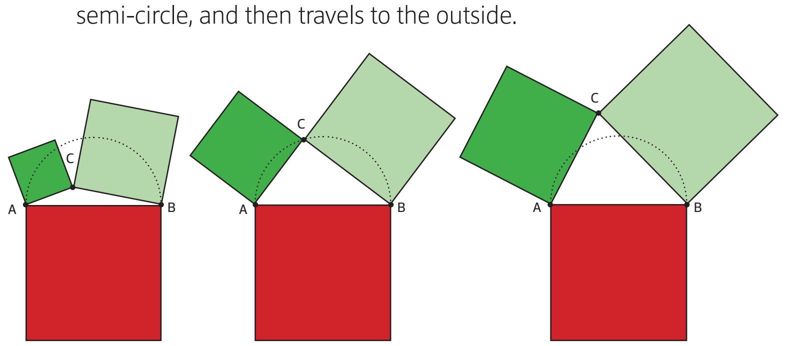
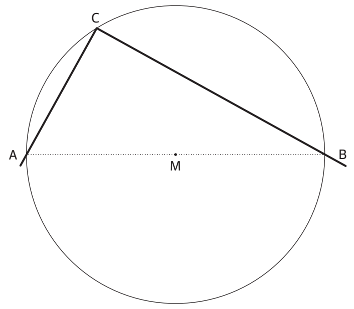
Bonanza

Thales – trailblazer for Pythagoras

Point C travels upwards from the inside to the outside of the semi-circle that has been drawn on the line section AB. The red square stays stable all the time – watch the green squares. Point C is located at the inside of the semi-circle, then on the semi-circle, and then travels to the outside.



1. Measure the shapes any way you want. What remains constant between the cases, what changes? What is special about the middle image?



2. Select a few different positions for C on the semi-circular arc. Each time, connect C with A and B. Measure the angle between the sides running from C to A and B. What do you find?

3. With your measurements, you have corroborated the Thales theorem, but you have not yet proven it. Why not?

4. The sides next to the right angle (CA and CB) are called 'catheti' (singular: 'cathetus'). The side opposite to the right angle (AB) is called 'hypotenuse'. Search the internet for these terms, which were originally Greek.

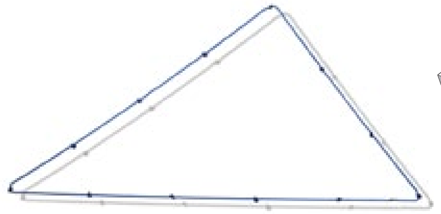
Pythagoras' theorem.

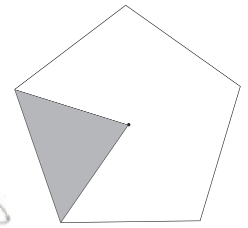
5. The converse of this theorem is used often. How can it be formulated?

Right triangles with integer side lengths are called **Pythagorean** or **Egyptian** triangles.

6. Find other Pythagorean triangles! There is certainly an infinite number of them - why?

In classical antiquity, a rope with twelve knots – as in this model – was used to mark out right angles in construction and in surveying.





7. Using the rope and writing material provided, form a right triangle ('carpenter triangle'), an isosceles triangle, a square, a rhombus, and approximately, the main triangle of a regular pentagon (as shown).

The folded handkerchief

8. Fold a square handkerchief so that another square is formed. Is there more than one way to do it?

9. Is there a smallest and a largest square?

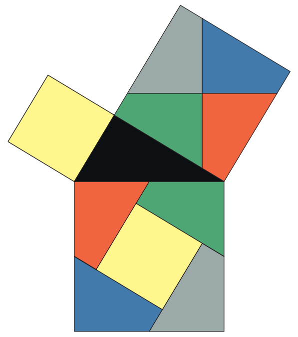
10. Mathematically speaking, the question about the size of the squares and their mutual relationships is an exiting one. What can you find?

By the way: if you use paper instead of a handkerchief, it looks even better.

Proven in multiple ways: the Pythagorean theorem.

There are more than 100 geometrical and algebraic proofs for Pythagoras' theorem – an indication of its outstanding importance.
Two proofs through dissection and transportation:

➲ **Paddlewheel proof by Henry Perigal (1801-1898)**
The dissected figure consists of the smallest possible number of five pieces and is symmetrical.



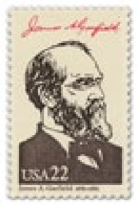
11. Develop and un-develop this model. What do you notice?

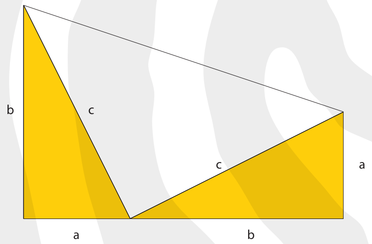
12. What is the core idea of the proof?

13. What does the Perigal configuration look like in a right isosceles triangle?

➲ **Garfield supports Pythagoras**

14. Suitable positioning of the four (yellow) right triangles in the empty field to the right results in an interesting square. Why? Do you recognise anything in this arrangement? Do you recognise anything already known from the handkerchief-folding exercise?





15. Calculate the sum of the areas of the three triangles from the Garfield proof and the area of the trapeze. Show that since the approaches are equal, the result is:
$a^2 + b^2 = c^2$

In the Pythagorean forest

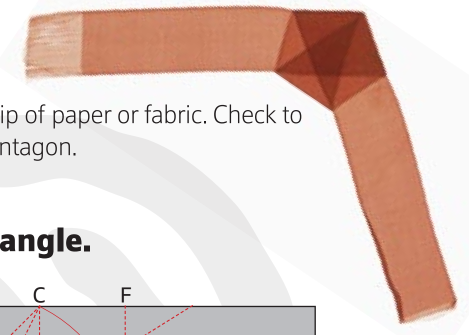
16. Why do equally tall 'firs' stand on a (quarter) circle?

17. What is special about the coordinates of the points on the 30°, 45°, and 60° radial lines?

The 'firs' on the *x*-axis and on the *y*-axis are special in that they are easy to calculate. How does this give an idea about the next question?

18. Why do 'fir heights' along a radial ray increase in a *linear* way?

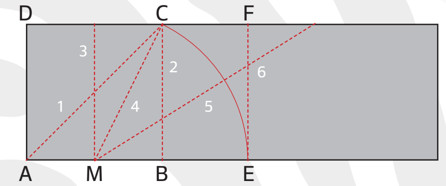
The golden section.



19. Bind a simple flat knot into a strip of paper or fabric. Check to see that the result is a regular pentagon.

Paper folding: golden rectangle.

A golden rectangle is created by simply folding a strip of paper. The square ABCD, which is created in the process, ultimately leads to the line MC, whose length contains the key number √5.

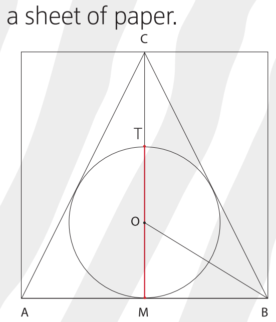


20. Fold lines 1 to 6 in sequence. When creating fold number 5 (as the bisection of the angle CMB), C touches the opposite rim of the paper strip at E. Create further patterns with the puzzle from golden rectangles.

The triangle ABC is embedded in the square with the side AB. It turns out that T sections the line MC at the golden section.

21. Construct point T by 'merely' folding a sheet of paper.

Two hints:
You only need the diameter of the circle. BO is the bisection of the (tri)angle ABC.



According to Le Corbusier, a man with his arm raised has the ideal measurements:

navel height:	113 cm
height to top of head:	183 cm
finger tips of raised arm:	226 cm

22. How about your measurements?

Study the front and rear side of a 10 Swiss franc bill and find more information by looking on the internet. In certain arrangements of leaves, the Lucas sequence 2, 1, 3, 4, 7, 11, 18,… is used as a description.

23. What is the rule for forming this sequence?

24. Every Lucas number from 3 onwards is the sum of two Fibonacci numbers. Check this statement against a few examples.

25. Does any tendency show in the quotients of neighbouring Lucas numbers? What do you suppose?

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